

# Can brains perform second-order optimization?

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## Abstract

**In ecological setup, a wide variety of organisms search over space to obtain reward using information obtained via multiple senses. In the simplest scenario of *scalar search*, a single quantity, e.g. concentration of a chemoattractant, is measured at different locations. Though gradient is considered a crucial component of scalar search, whether organisms rely solely on the gradient is unknown. We hypothesized that scalar search benefits from information other than gradient, including curvature (second-order derivatives) and long-term memory information integration. To test our hypothesis, we devised an information foraging task. In our task, human subjects control a circular avatar to find a peak of the contour by making brief fixations. They were rewarded when they approached the peak within the predefined maximum number of fixations. In our preliminary data, observed search trajectories deviated from what is expected from the gradient-based search, suggesting that the subjects utilized information beyond the gradient. We also manipulated the perception and action components of the task to examine the sensitivity of the adopted strategies to variations of the task design.**

**Keywords:** foraging; Lévy walks; gradient; second-order optimization; directed search;

## Introduction and Background

Arguably, one of the main functions of any naturally evolved cognitive system is to locate resources. Humans and other primates benefit from multiple senses, long-term memory, and inductive and deductive reasoning, that allow them to efficiently explore and utilize their environment. This complicated machinery, however, evolved from and still rely on many simple algorithms that had once been used by our ascendants. It is therefore important to identify features of search processes

that appear in a wide range of organisms, as they may shed light on the evolutionary preserved strategies adopted by life in general and our brains in particular. Optimality theory can then be incorporated as a guiding principle to point where to look for such strategies or to explain why they might have appeared (Parker & Smith, 1990) (see (Gould & Lewontin, 1979) for a discussion of the limitations of such an approach).

In a relatively recent example, non-gaussian, superdiffusive patterns of movement have been observed in a number of organisms, including bacteria (Korobkova, Emonet, Vilar, Shimizu, & Cluzel, 2004; Ariel et al., 2015), wide range of flying (G. M. Viswanathan et al., 1996; A. M. Reynolds et al., 2007; A. M. Reynolds & Frye, 2007) and swimming animals (Sims et al., 2008), human mobility patterns (Brockmann, Hufnagel, & Geisel, 2006; Raichlen et al., 2014), and even in the patterns of human gaze (Brockmann & Geisel, 2000), see (G. M. Viswanathan, Da Luz, Raposo, & Stanley, 2011; A. Reynolds, 2015; A. M. Reynolds, 2018) for more comprehensive lists. Such behavior can be modelled with Lévy flights/walks (LFs) (Shlesinger & Klafter, 1986; Zaburdaev, Denisov, & Klafter, 2015). LFs have been shown to offer an efficient random search strategy in many scenarios (G. M. Viswanathan et al., 1999; Bartumeus, Catalan, Fulco, Lyra, & Viswanathan, 2002; Bartumeus, da Luz, Viswanathan, & Catalan, 2005; Kusmierz, Majumdar, Sabhapandit, & Schehr, 2014; Kuśmierz & Gudowska-Nowak, 2015) (but see (James, Plank, & Brown, 2008; Palyulin, Chechkin, & Metzler, 2014) for counterexamples), which has led to the hypothesis that such non-diffusive random walks appear as a result of natural selection (G. Viswanathan, Raposo, & Da Luz, 2008). Regardless whether this is true or not, the possible mechanisms behind the appearance of LFs are of great interest and multiple models showing different mechanisms of emergence of LFs have been put forward to date, see (G. M. Viswanathan et al., 2011; A. Reynolds, 2015) for reviews.

Importantly, even the simplest organisms have the ability to act and react to environmental signals, leading to

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a non-random, directed search. Examples include phototaxis (Jékely et al., 2008), rheotaxis (Marcos, Fu, Powers, & Stocker, 2012), and chemotaxis (Adler, 1966; Keller & Segel, 1971). Although in principle non-random, such strategies can lead to random-looking movement patterns in a complex or fluctuating environments. For instance, in the absence of chemoattractant gradients, *E. coli* bacteria is still moving, apparently randomly (Berg & Brown, 1972). Most models of chemotaxis assume that it is driven by estimates of gradient of the concentration of the chemoattractant (or chemorepellent). Recently it has been shown that noisy second-order optimization (Newton method) should invariably lead to LFs signatures (Kuśmierz & Toyozumi, 2017). Such combination of directed and random search is expected to form an efficient search strategy due to the following two factors. (1) Newton method is optimal close to the target and (2) LFs are efficient far from the target, where no information about its location is provided.

Here we report on the development of an experiment by which we intend to test whether such more complicated higher order derivative calculations are implemented by primate brains. In our framework the subjects can efficiently use just local information but also to integrate information across time steps. This allow us to investigate the relative cost and efficiency of perceptual and memory driven search. The questions that we plan to address include: (a) What are the strategies adopted by subjects? (b) In particular, is second-order optimization being used? (c) Do Lévy walks emerge? (d) How do features of the search depend on the urgency and information quality? In the following we present the framework that we are developing to address these questions and results of a preliminary study performed on human subjects.

## Method

### The experimental design

The preliminary result includes 3 human subjects. The goal of subjects at each trial is to move a self-representing avatar closer than 100 pixel from the peak of mountain (target). The subjects controlled the yellow circular avatar at screen by joystick. The peak of mountains is not revealed, but only gray screen was shown. Only when the subjects hold the joystick for 300 ms at one location, the gradient information (a small circular view of the landscape) is revealed.

The information about the location of the target is provided by the visible pixels intensity through the scalar function

$$f(\mathbf{x}) = \frac{1}{1 + (\mathbf{x} - \mu)^T \mathbf{A} (\mathbf{x} - \mu)}, \quad (1)$$

where the location vector  $\mu$  and matrix  $\mathbf{A}$  are generated randomly at each trial. Eigenvalues of  $\mathbf{A}$  are independent and uniform in the interval (100, 700), the position of the target is uniform over the landscape, and a random rotation is applied to  $\mathbf{A}$  so that no particular direction is statistically preferred.

<sup>1</sup>Hessian  $\mathbf{H}$  calculated at the location of the maximum  $\mu$  is related to  $\mathbf{A}$  through the relation  $\mathbf{H} = -2\mathbf{A}$ .

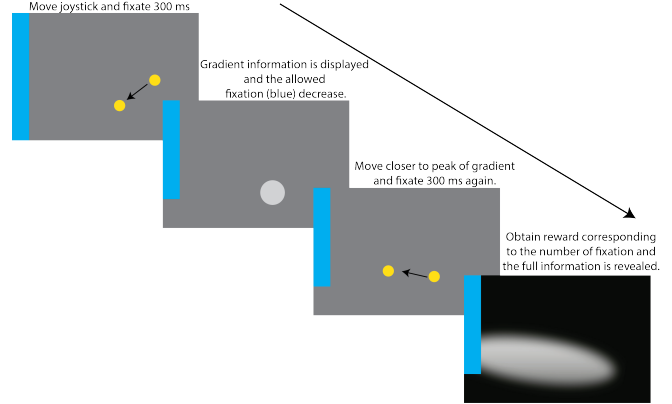


Figure 1: The experimental design. Subjects control the circular avatar (yellow) at the screen. The number of allowed fixation was shown as bar at the left edge of the screen (blue). Once the fixation has been made for 300 ms by holding joystick still, then the local information about the landscape is revealed for 500 ms. Each block has different number of maximum fixation, speed of avatar movement, and amount of information displayed once fixated.

The session for each subject is consisting of 8 blocks and each block has 30 trials (total 240 trials per each subject). The three major parameters changed at each block are 1) amount of information revealed once fixating (35 pixels vs. 50 pixels), 2) maximum number of fixation (5 vs. 8), and 3) maximum speed of avatar (12 pixels/sec vs. 24 pixels/sec). These parameters varied to examined the effect of perception, memory, and action, respectively. The block sequence was randomized.

The score they obtain at each trial is intended to incentivize the subjects. The whole landscape was revealed to subjects regardless of their results. To prevent subject being less motivated, we impose 30 seconds time out limits. The block changes were indicated by written message but the parameter change has not informed.

## Result

### Human subjects deploy other information than the gradient

Our preliminary results suggest that humans can employ information beyond gradient, see Fig. 2. However, this can be attributed to either integration of information over time or memory of environment. Thus, we selected only the first fixation of each trial and repeated our analysis. our result is also true if only first fixation at each trial are considered, removing the influence of memory. This result suggests that subjects might have used curvature information in their search. More data is needed to dissect the exact strategy and its sensitivity to the task design.

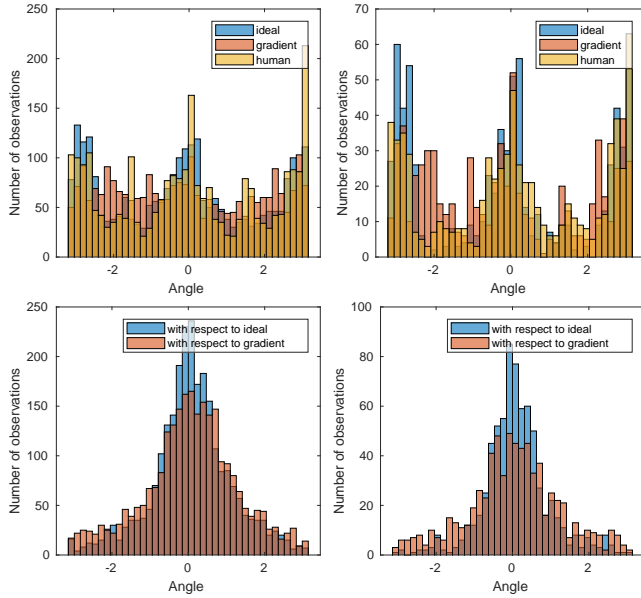


Figure 2: Histograms of angles of vectors representing [top]: (1) “ideal” steps, i.e. connecting a given fixation point with the position of the target, (2) gradient calculated from the pixels visible at the fixation points, (3) actual steps generated by human subjects, i.e. connecting consecutive fixation points, [bottom]: angles between actual steps generated by the subjects and (1) ideal and (2) gradient steps. Left plots represent all data points, whereas right plots show data limited to the very first step in each trial, effectively removing memory effects. Even in this case subjects performed significantly better than possible by assuming that search is driven only by gradient.

### Lack of evidence for Lévy walks

Our results suggest the lack of evidence for Lévy walk pattern, see Fig. 3. To examine the pattern of search, we calculated jump length at each movement. If search pattern mimics Lévy walk, then the jump length distribution should be a power-law that spans a few orders of magnitude. However, we did not find any signatures of such strongly long-tailed distribution. Retrospectively, it is easy to understand why we should not expect such power-law distributions to appear in our task. The circular fixation point sets a minimal jump length that makes sense for the subject, which is of the order of 100 pixels. On the other hand, the movement is constrained by the screen size and thus the maximum possible jump length is of the order of 1000 pixels. There is simply not enough orders of magnitude to span in order to create a power law.

### Discussion

Here, we present an empirical paradigm that examines the source of information being used for search. We find that the human subjects do not exclusively rely on the first order information. By separating the initial step only, we show that memory and information integration are not necessary for the

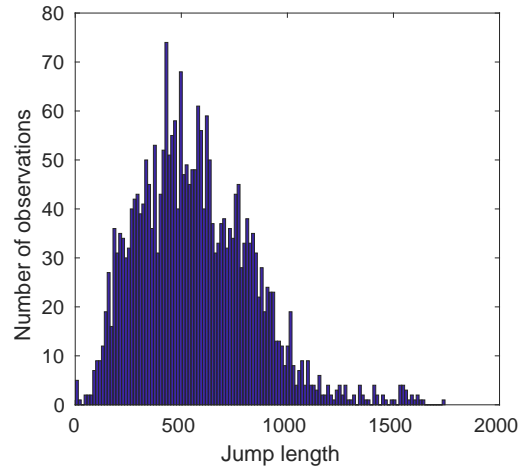


Figure 3: Histogram of the jump length distribution across all subjects and trials.

appearance of the effect. Although we modified conditions to isolate the factors influencing search patterns (e.g. perception or action), we did not see significant conditional difference. Instead, we observed that the success rate significantly differed between conditions with maximum number of fixation points, which is a proxy for the urgency. Thus, two goals for the future study are: splitting our result by the urgency and creating empirical conditions where perception or action parameters differ saliently so that subjects can take distinctive search strategy. By this device, we expect to examine the attributing factor for search strategy. Furthermore, we expect to add the gaze and pupil data so that we can exhibit effect of attention.

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